Problem 1.2

Vector algebra 2*
Given two vectors $\mathbf{A} = (3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 5\hat{\mathbf{k}})$ and $\mathbf{B} = (6\hat{\mathbf{i}} - 7\hat{\mathbf{j}} + 4\hat{\mathbf{k}})$ find:
(a) $\mathbf{A}^2$;   (b) $\mathbf{B}^2$;   (c) $(\mathbf{A} \cdot \mathbf{B})^2$.

Solution

To find $\mathbf{A}^2$, we take the dot product of $\mathbf{A}$ with itself. We find $\mathbf{B}^2$ in the same way. The dot product is calculated by multiplying the respective components and adding them.

Part (a)

$$\mathbf{A}^2 = \mathbf{A} \cdot \mathbf{A} = (3)(3) + (-2)(-2) + (5)(5) = 38$$

Part (b)

$$\mathbf{B}^2 = \mathbf{B} \cdot \mathbf{B} = (6)(6) + (-7)(-7) + (4)(4) = 101$$

Part (c)

To find $(\mathbf{A} \cdot \mathbf{B})^2$, calculate $\mathbf{A} \cdot \mathbf{B}$ and then square the result.

$$\mathbf{A} \cdot \mathbf{B} = (3)(6) + (-2)(-7) + (5)(4) = 52$$

$52^2 = 2704$. Therefore,

$$(\mathbf{A} \cdot \mathbf{B})^2 = 2704.$$